AP Calculus AB

Review Week 3

Applications of Derivatives

Advanced Placement AAP Review will be held in **room 315** and **312** on Tuesdays and Thursdays.

The week of April 6th we will be reviewing **Applications of Derivatives**

The session will begin in room 315 with a brief review of the weekly topic.

Instruction will be from 3:00 pm to 3:15 pm

Once we have reviewed the topic you may begin practicing the questions in your review packet.

Answers will be posted in room 315 and 312 all week and will be posted on line after 3:00 pm on Friday the week of review.

If you have difficulty with a question look at the detailed answer postings BEFORE you ask your teacher for help.

Get a hint….**<u>DON'T COPY THE ANSWER!!! THAT IS NOT HELPFUL!!</u>**

When you have completed a question…**<u>REFLECT!!!!</u>** Ask yourself what skill you used to solve that problem and write that down!!

Once we have completed the weekly review, keep it to study from as we get closer to the exam.

Applications of Derivatives

Brief Review

It is important in Calculus to study the application of derivatives to the understanding of the characteristics of the graph of a function.

MAX and MIN

Peaks and valleys…usually occur when the derivative = ZERO (horizontal tangent) or the derivative is undefined (POINTY TOP OR BOTTOM)…the Extreme Value Theorem reminds you to check the endpoints when you are considering a function on a closed interval.

INCREASING or DECREASING

Increasing when the derivative is POSITIVE or ABOVE the x-axis. Decreasing when the derivative is NEGATIVE or BELOW the x-axis.

Concave UP or DOWN

- **When the DERIVATIVE is INCREASING the graph of f(x) is Concave UP.**
- **When the DERIVATIVE is DECREASING the graph of f(x) is Concave DOWN.**

Inflection points occur when we change concavity.

There is a first derivative test for Max and Mins and a second derivative test.

There are 3 Theorems you should know the IVT, EVT, MVT.

You should know how to do OPTIMIZATION and RELATED RATE problems.

Extreme Value Theorem:

- If a function $f(x)$ is continuous on a closed interval $[a, b]$, then $f(x)$ has both a maximum and minimum value on $[a, b]$.

Mean Value Theorem:

If a function *f(x)* is continuous on a closed interval [a,b] and differentiable on an open interval (a,b), then at least one number c ∊ (a,b) exists such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Intermediate Value Theorem:

If f is continuous on a closed interval $[a, b]$, and c is any number between $f(a)$ and $f(b)$ inclusive, then there is at least one number x in the closed interval such that $f(x) = c$.

2nd Derivative Test

The second derivative may be used to determine local extrema of a function under certain conditions. If a function has a critical point for which $f'(x) = 0$ and the second derivative is positive at this point, then f has a local minimum here. If, however, the function has a critical point for which $f'(x) = 0$ and the second derivative is negative at this point, then f has local maximum here. This technique is called **Second Derivative Test for Local Extrema.**

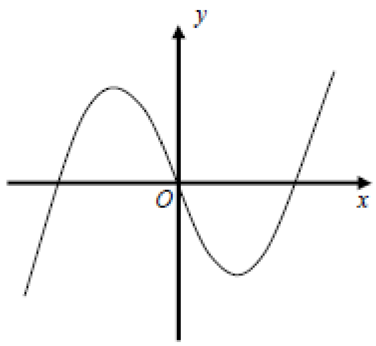
Fails if…

(1) $f'(x) = 0$ and $f''(x) = 0$

(2) $f'(x) = 0$ and $f''(x)$ does not exist

(3) $f'(x)$ does not exist

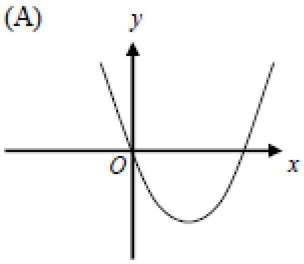
2008 No Calculator



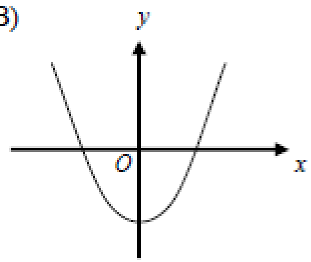
Graph of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

(A)



(B)



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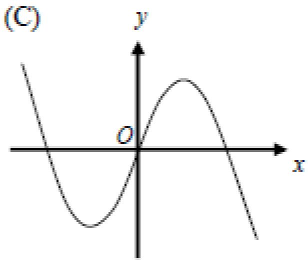

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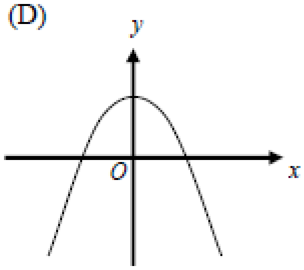

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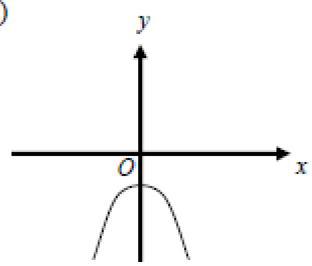


x	0	1	2	3
$f''(x)$	5	0	−7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

(A) 0 only

(B) 3 only

(C) 0 and 6 only

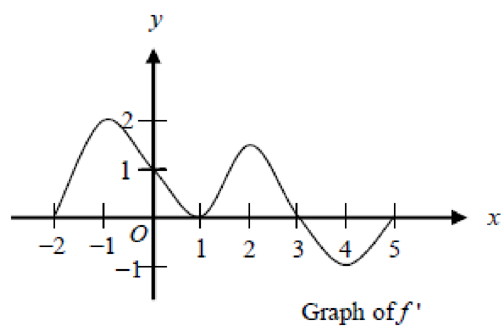
(D) 3 and 6 only

(E) 0, 3, and 6

24. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

2008 No Calculator



76. The graph of f', the derivative f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

(A) $[-2, 1]$ only

(B) $[-2, 3]$

(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$

78. The first derivative of the function f is defined by $f'(x) = \sin(x^3 - x)$ for $0 \le x \le 2$. On what interval(s) is f increasing?

(A) $1 \le x \le 1.445$

(B) $1 \le x \le 1.691$

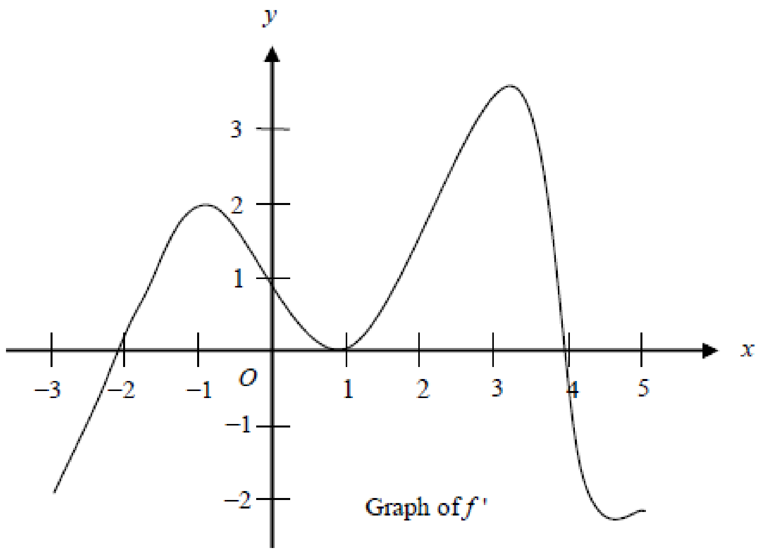
(C) $1.445 \le x \le 1.875$

(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$

(E) $0 \le x \le 1$ and $1.691 \le x \le 2$

80. The derivative of the function f is given by $f'(x) = x^2 \cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five



84. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

(A) -2 only

(B) 1 only

(C) 4 only

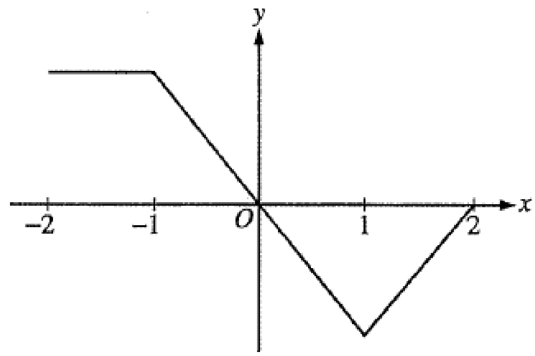
(D) -1 and 3 only

(E) -2, 1, and 4

88. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

2003 No Calculator



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f ?

(A) f is decreasing for $-1 \leq x \leq 1$.

(B) f is increasing for $-2 \leq x \leq 0$.

(C) f is increasing for $1 \leq x \leq 2$.

(D) f has a local minimum at $x = 0$.

(E) f is not differentiable at $x = -1$ and $x = 1$.

12. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

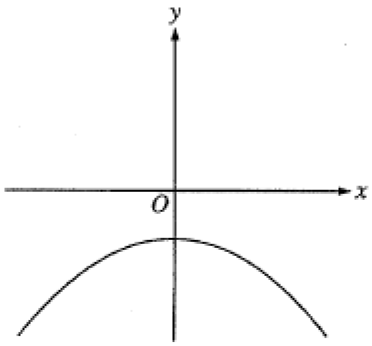
(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

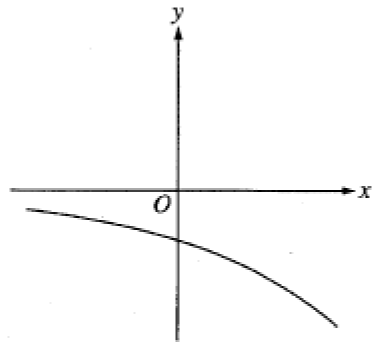
(E) $\frac{dV}{dt} = k\sqrt{V}$

10. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f ?

(A)



(B)



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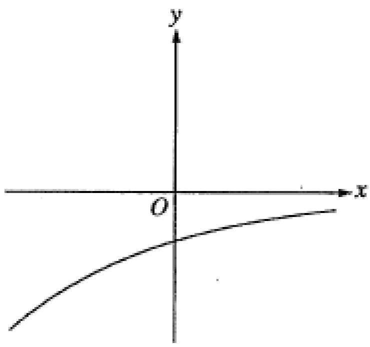
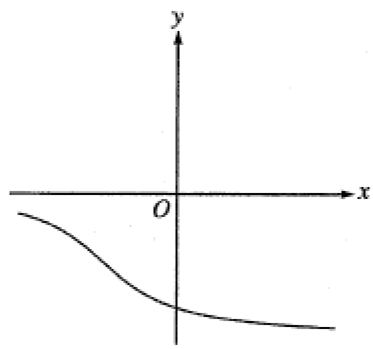
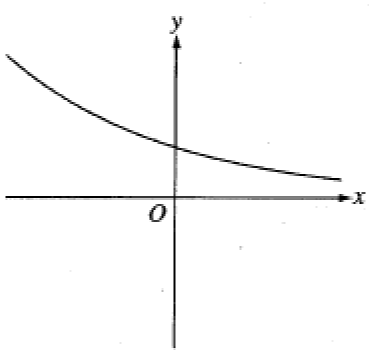


(D)



(E)



15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only

(B) $(-\infty, 0)$

(C) $[-1, 0)$ only

(D) $(0, \sqrt[3]{2}]$

(E) $[\sqrt[3]{2}, \infty)$

17. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

18. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

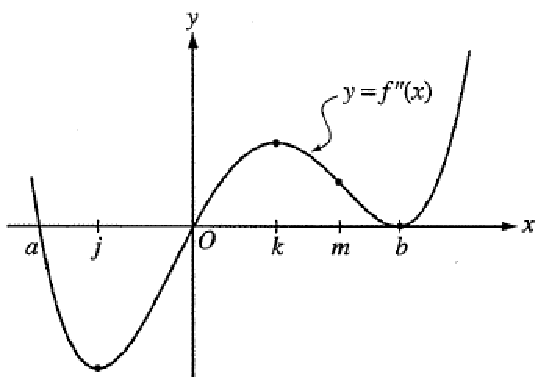
(A) $-2 \le x \le 2$ only

(B) $-1 \le x \le 1$ only

(C) $x \ge -2$

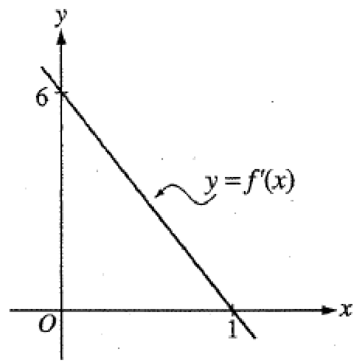
(D) $x \ge 2$ only

(E) $x \le -2$ or $x \ge 2$



21. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only (B) 0 and m only (C) b and j only (D) 0, a, and b (E) b, j, and k



22. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 (D) 8 (E) 11

25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

28. Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

(A) 15 (B) 18 (C) 21 (D) 24 (E) 27

2003 Calculator

78. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) 0.04π m^2/sec

(B) 0.4π m^2/sec

(C) 4π m^2/sec

(D) 20π m^2/sec

(E) 100π m^2/sec

81. Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One (B) Two (C) Three (D) Four (E) Five

87. The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$. What is the x-coordinate of the inflection point of the graph of f ?

(A) 1.008 (B) 0.473 (C) 0 (D) −0.278 (E) The graph of f has no inflection point.

90. For all x in the closed interval $[2, 5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

(A)

x	$f(x)$
2	7
3	9
4	12
5	16

(B)

x	$f(x)$
2	7
3	11
4	14
5	16

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

(E)

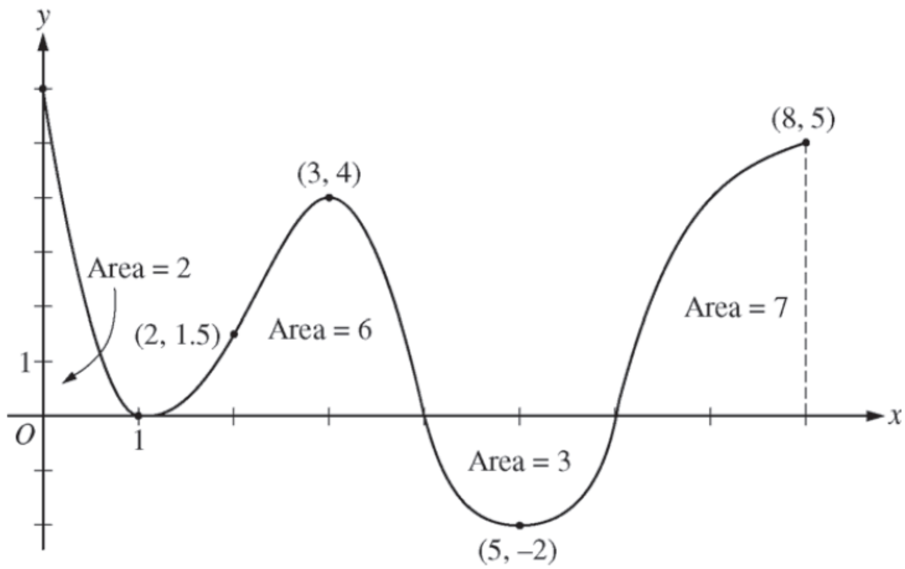
x	$f(x)$
2	16
3	13
4	10
5	7

Free Response 2013 #1

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

Free Response 2013 NON CALCULATOR #4



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

Free Response NON Calculator 2008 #3

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

Free Response NON-Calculator 2007 #3

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

Free Response NON-Calculator 2007 #5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.