Vocabulary

5-7 Inequalities in Two Triangles

Vocabulary

Review

Circle the included angles in each diagram.

1. A
   B
   C

2. Y
   X
   W
   Z

In Exercises 3–5, cross out the group of values that does not satisfy the Comparison Property of Inequality.

3. \( a = 3, b = 3, c = 0 \)
   \( a = 6, b = 4, c = 2 \)

4. \( a = 11, b = 3, c = 8 \)
   \( a = 1, b = 2, c = 3 \)

5. \( a = 8, b = 3, c = 5 \)
   \( a = 8, b = 5, c = 4 \)

Write a number so that each group satisfies the Comparison Property of Inequality.

6. \( a = 2, b = 0, c = 2 \)

7. \( a = 9, b = 8, c = 1 \)

8. \( a = 3, b = 1, c = 2 \)

Vocabulary Builder

hinge (noun, verb) hinj

Definition (noun): A hinge is a device on which something else depends or turns.

Definition (verb): To hinge upon means to depend on.

Use Your Vocabulary

Circle the correct form of the word hinge.

9. Everything hinges on his decision. Noun / Verb

10. The hinge on a gate allows it to swing open or closed. Noun / Verb

11. Your plan hinges on your teacher’s approval. Noun / Verb

12. The lid was attached to the jewelry box by two hinges. Noun / Verb
13. Use the triangles at the right to complete the table.

<table>
<thead>
<tr>
<th>Theorem</th>
<th>If...</th>
<th>Then...</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-13: Hinge Theorem</td>
<td>( m_\angle A &gt; m_\angle X )</td>
<td>( BC &gt; YZ )</td>
</tr>
<tr>
<td>5-14: Converse of the Hinge Theorem</td>
<td>( BC &gt; YZ )</td>
<td>( m_\angle A &gt; m_\angle X )</td>
</tr>
</tbody>
</table>

14. Explain why Theorems 5–13 and 5–14 are also called the SAS and SSS Inequality Theorems.

15. Use information in the diagram to complete each statement.

The included angle in \( \triangle LMN \) is \( \angle M \).

The included angle in \( \triangle OPQ \) is \( \angle P \).

16. Circle the side opposite the included angle in \( \triangle LMN \). Underline the side opposite the included angle in \( \triangle OPQ \).

\( \overline{LM} \) \( \overline{MN} \) \( \overline{OP} \)

17. Use the Hinge Theorem to complete the statement below.

\( m_\angle M > m_\angle P \), so \( LN > QO \)
Problem 3 Using the Converse of the Hinge Theorem

Got It? What is the range of possible values for \( x \) in the figure at the right?

18. From the diagram you know that the triangles have two pairs of congruent corresponding sides, that \( LM < 90 \), and that \( m \angle N = 90 \).

Complete the steps and justifications to find upper and lower limits on \( x \).

19. \( m \angle K < 90 \)  
   Converse of the Hinge Theorem
   \[ 3x + 18 < 90 \]  
   Subtract \( 18 \) from each side.
   \[ 3x < 72 \]  
   Divide each side by \( 3 \).
   \[ x < 24 \]

20. \( m \angle K > 0 \)  
   The measure of an angle of a triangle is greater than 0.
   \[ 3x + 18 > 0 \]  
   Subtract \( 18 \) from each side.
   \[ 3x > -18 \]  
   Divide each side by \( 3 \).
   \[ x > -6 \]

21. Write the two inequalities as the compound inequality \(-6 < x < 24\).

Problem 4 Proving Relationships in Triangles

Got It? Given: \( m \angle MON = 80 \); \( O \) is the midpoint of \( \overline{LN} \).

Prove: \( LM > MN \)

22. Write a justification for each statement.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( m \angle MON = 80 )</td>
<td>1) ( \text{Given} )</td>
</tr>
<tr>
<td>2) ( m \angle MON + m \angle MOL = 180 )</td>
<td>2) ( \text{def. of supp. \angle s} )</td>
</tr>
<tr>
<td>3) ( 80 + m \angle MOL = 180 )</td>
<td>3) ( \text{Substitution} )</td>
</tr>
<tr>
<td>4) ( m \angle MOL = 100 )</td>
<td>4) ( \text{Subtraction} )</td>
</tr>
<tr>
<td>5) ( \overline{LO} \cong \overline{ON} )</td>
<td>5) ( \text{def. of midpoint} )</td>
</tr>
<tr>
<td>6) ( \overline{MO} \cong \overline{MO} )</td>
<td>6) ( \text{reflexive prop. of \overline{}} )</td>
</tr>
<tr>
<td>7) ( m \angle MOL &gt; m \angle MON )</td>
<td>7) ( 100 &gt; 80 )</td>
</tr>
<tr>
<td>8) ( LM &gt; MN )</td>
<td>8) ( \text{hinge theorem} )</td>
</tr>
</tbody>
</table>
Lesson Check • Do you know HOW?

Write an inequality relating $FD$ and $BC$.

In Exercises 23–26, circle the correct statement in each pair.

23. $AC = EF$  $AC > EF$
24. $AB > ED$  $AB = ED$
25. $m\angle BAC > m\angle FED$  $m\angle BAC < m\angle FED$
26. By the Hinge Theorem, you can relate $FD$ and $BC$.

By the Converse of Hinge Theorem, you can relate $FD$ and $BC$.

27. Write an inequality relating $FD$ and $BC$.

$FD > BC$

Lesson Check • Do you UNDERSTAND?

Error Analysis  From the figure at the right, your friend concludes that $m\angle BAD > m\angle BCD$. How would you correct your friend’s mistake?

Write T for true or F for false.

28. $AB = CD$  $F$  29. $AD = CB$  $T$  30. $BD = BD$

31. Your friend should compare $\overline{AD}$ and $\overline{BC}$.
32. The longer of the two sides your friend should compare is $\overline{AD}$.
33. How would you correct your friend’s mistake? Explain.

$m\angle BAD > m\angle CCB$D

Math Success

Check off the vocabulary words that you understand.

☐ exterior angle   ☐ comparison property of inequality   ☐ Hinge Theorem

Rate how well you can use triangle inequalities.

<table>
<thead>
<tr>
<th>Need to review</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>Now I get it!</th>
</tr>
</thead>
</table>